CHAPTER ONE
Fluid Fundamentals

1.1 FLUID PROPERTIES

1.1.1 Mass and Weight

Mass, m, is a property that describes the amount of matter in an object or fluid. Typical units are slugs in U.S. customary units, where one slug is equivalent to 32.2 pounds-mass (lbm), and kilograms (kg) in the International System of Units (SI).

Weight, W, of an object or fluid is defined from Newton's second law of motion (i.e., $\vec{F} = m\vec{a}$) as the product of mass and gravitational acceleration, g, or

$$W = mg \tag{1-1}$$

At the earth's surface, g is equal to 32.2 ft/s^2 in U.S. customary units and 9.81 m^2/s in SI units.

<u>Example</u>: Compute the mass of an object weighing 1 pound-force (lbf).

Solution:

Mass is computed by rearranging Equation 1-1.

$$m = \frac{W}{g} = \frac{1\ lb}{32.2\ ft/s^2} = \frac{1}{32.2}\ slug$$

Converting from slugs to pounds-mass,

$$\frac{1}{32.2}\ slug \times \frac{32.2\ lbm}{slug} = 1\ lbm$$

Thus, a weight of 1 lbf is equivalent to 1 lbm at the earth's surface.

<u>Example</u>: Evaluate the weight of a 1-kg object at the earth's surface.

Solution:

Again, from Equation 1-1,

$$W = mg = (1\,kg)(9.81\,m/s^2) = 9.81\,kg \cdot m/s^2$$

The SI unit of $kg \cdot m/s^2$ is commonly referred to as a Newton (N). This is equivalent to the force required to accelerate 1 kg of mass at a rate of 1 m/s^2.

1.1.2 Mass Density

Density, ρ, is the mass of fluid per unit volume, $\forall$. It varies as a function of temperature, as shown in Table 1-1. For most practical purposes, however, density is assumed to be constant, and a typical value for water or wastewater applications is 1.94 slugs/ft^3, or 1,000 kg/m^3.

Table 1-1: Physical properties of water

Temp.	Density (ρ)	Specific weight (γ)	Dynamic viscosity (μ)	Kinematic viscosity (ν)	Vapor pressure (absolute) (p_v)
(°F)	(slugs/ft^3)	(lb/ft^3)	(lb-s/ft^2)	(ft^2/s)	(psi)
40	1.94	62.43	3.23×10^{-5}	1.66×10^{-5}	0.122
50	1.94	62.40	2.73×10^{-5}	1.41×10^{-5}	0.178
60	1.94	62.37	2.36×10^{-5}	1.22×10^{-5}	0.256
70	1.94	62.30	2.05×10^{-5}	1.06×10^{-5}	0.363
80	1.93	62.22	1.80×10^{-5}	9.30×10^{-6}	0.506
100	1.93	62.00	1.42×10^{-5}	7.39×10^{-6}	0.949
120	1.92	61.72	1.17×10^{-5}	6.09×10^{-6}	1.69
140	1.91	61.38	9.81×10^{-6}	5.14×10^{-6}	2.89
(°C)	(kg/m^3)	(N/m^3)	(N-s/m^2)	(m^2/s)	(N/m^2)
0	1,000	9,810	1.79×10^{-3}	1.79×10^{-6}	611
10	1,000	9,810	1.31×10^{-3}	1.31×10^{-6}	1,230
20	998	9,790	1.00×10^{-3}	1.00×10^{-6}	2,340
30	996	9,771	7.97×10^{-4}	8.00×10^{-7}	4,250
40	992	9,732	6.53×10^{-4}	6.58×10^{-7}	7,380
50	988	9,693	5.47×10^{-4}	5.53×10^{-7}	12,300
60	983	9,643	4.66×10^{-4}	4.74×10^{-7}	20,000

<u>Example</u>: Determine the mass of 2 ft^3 of water.

Solution:

Based on the definition of mass density,

$$m = \rho\forall = \left(1.94\ \frac{slugs}{ft^3}\right)\left(2\ ft^3\right) = 3.88\ slugs$$

1.1.3 Specific Weight

Specific weight, γ, (see Table 1-1) represents the weight of a fluid per unit volume. A typical value for water is approximately 62.4 lb/ft^3, or 9,810 N/m^3. Through Newton's second law, specific weight is related to mass density by

$$\gamma = \rho g \tag{1-2}$$

<u>Example</u>: Determine the specific weight and weight of 2 m^3 of a fluid having a mass density of 1,030 kg/m^3.

Solution:

From Equation 1-2,

$$\gamma = \rho g = \left(1{,}030\ \frac{kg}{m^3}\right)\left(9.81\ m/s^2\right) = 10{,}104\ N/m^3$$

and based on the definition of specific weight,

$$W = \gamma\forall = \left(10{,}104\ \frac{N}{m^3}\right)\left(2\ m^3\right) = 20{,}208\ N$$

1.1.4 Specific Gravity

When dealing with fluids other than water, it is common to express their specific weight or density relative to that of water at 4°C, or

$$S = \frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w} \tag{1-3}$$

where γ_w and ρ_w are 62.4 lbs/ft^3 and 1.94 slugs/ft^3 (9,810 N/m^3 and 1,000 kg/m^3), respectively, and the resulting dimensionless ratio, S, is known as specific gravity. For water, S is approximately 1.0. An estimate of S for wastewater is 1.0025, indicating that it is 0.25 percent heavier than water, a difference that is negligible for most applications. The specific gravity of sludge, however, can be as large as 1.05. A corresponding difference of this magnitude may be justifiably incorporated into hydraulic computations.

Example: Consider a 3-ft^3 volume of sludge weighing 192 lbs. Compute the specific gravity of the sludge.

Solution:

Based on the definition of specific gravity,

$$S = \frac{\gamma}{\gamma_w} = \frac{W}{\gamma_w \forall} = \frac{192\ lbs}{\left(62.4\ lbs/ft^3\right)\left(3\ ft^3\right)} = 1.03$$

1.1.5 Viscosity

Dynamic, or absolute, viscosity of a fluid, μ, (see Table 1-1) represents a fluid's ability to resist deformation caused by shear forces (i.e., friction) at a pipe or conduit wall. The rate of deformation, dV/dy, is related linearly to shear stress, τ, by Newton's Law of Viscosity, expressed as

$$\tau = \mu \frac{dV}{dy} \tag{1-4}$$

where V is flow velocity at a distance y from the conduit wall. The relationship shows that, for a given shear stress, highly-viscous fluids will exhibit less deformation than thin fluids. In application, a related term known as kinematic viscosity, ν, is commonly used. Kinematic and dynamic viscosities are related by

$$\nu = \frac{\mu}{\rho} \tag{1-5}$$

Typical units of are lb-s/ft^2 and N-s/m^2 for μ and ft^2/s and m^2/s for ν.

Example: The specific gravity and kinematic viscosity of a liquid are 1.4 and 3×10^{-4} m^2/s, respectively. Compute the liquid's dynamic viscosity.

Solution:

The density is first computed from specific gravity.

$$\rho_{liquid} = S_{liquid}\rho_w = 1.4 \times 1{,}000\ \frac{kg}{m^3} = 1{,}400\ \frac{kg}{m^3}$$

From Equation 1-5,

$$\mu = \nu\rho = \left(3 \times 10^{-4}\ \frac{m^2}{s}\right)\left(1{,}400\ \frac{kg}{m^3}\right) = 0.42\ \frac{N \cdot s}{m^2}$$

1.1.6 Pressure

Pressure is equivalent to a point or distributed force, *F*, applied normal to a surface, divided by the area, *A*, of that surface, or

$$p = \frac{F}{A} \tag{1-6}$$

where *p* is pressure, typically expressed in lbs/in^2 (psi) or N/m^2 (Pascal). Attention should be paid to whether values are expressed in absolute or gage units. Pressure in a complete vacuum is referred to as absolute zero, and any pressure referenced relative to this zero is called absolute pressure. In most applications, however, the variation between pressure in the fluid and local atmospheric pressure (i.e., ≈ 14.7 psi or 101 Pa absolute) is sought. In this case, the resulting fluid pressure is referred to as gage pressure, and

$$p_{gage} = p_{abs} - p_{atm} \tag{1-7}$$

Pressure at a point can also be expressed as a unit of head, or height, *h*, of a column of water or other fluid that it supports. The hydrostatic law states that

$$p = \gamma h \tag{1-8}$$

where *p* is relative to the pressure at the surface of the fluid. Using this relationship, it can be shown that 1 psi is the equivalent of a 2.31-ft column of water.

Example: Determine the pressure at a depth of 6 ft in (a) an open tank of water and (b) a closed tank in which the surface is pressurized to 200 psi (absolute).

Solution:

For an open tank,

$$p = \gamma h = \left(62.4\ lbs/ft^3\right)\left(6\ ft\right) \times \frac{1\ ft^2}{144\ in^2} = 2.6\ psi\ (gage)$$

Assuming an atmospheric pressure of 14.7 psi (absolute), this is equivalent to

$$p_{abs} = p_{gage} + p_{atm} = 2.6 + 14.7 = 17.3\ psi\ (absolute)$$

For a closed tank and a surface pressure of 200 psi (absolute), the pressure at a depth of 6 ft is 2.6 + 200, or 202.6 psi (absolute). The corresponding gage pressure is

$$p_{gage} = p_{abs} - p_{atm} = 202.6 - 14.7 = 187.9\ psi\ (gage)$$

1.1.7 Vapor Pressure

Vapor pressure is the pressure at which a liquid will boil. Since vapor pressure varies with temperature as indicated in Table 1-1, boiling can be caused by raising the fluid temperature to the point where vapor pressure is equal to the atmospheric pressure (i.e., 212°F at 14.7 psia). However, boiling can also occur without a change in temperature if the pressure in the fluid is reduced to its vapor pressure, typically caused by a significantly increased flow velocity. In these cases, vapor bubbles form in the area of low pressure and subsequently collapse in downstream areas of higher pressure. The impact force created by the collapse can cause damage to surrounding surfaces and mechanical parts. This process, known as cavitation, is a major concern at the suction side of pumping systems.

Example: 1-ft^3 of water at 80°F is placed in a 2-ft^3 airtight container. If air is gradually pumped from the container, what reduction below atmospheric pressure (i.e., 14.7 psi) is required for the water to boil.

Solution:

From Table 1-1, the vapor pressure of water at 80°F is 0.506 psi (absolute). Thus, pressure must be reduced by 14.7 – 0.506, or 14.2 psi for boiling to occur.

1.2 FLOW CLASSIFICATIONS

1.2.1 Pressurized vs. Open-Channel Flow

Flow in water and wastewater systems is either pressure driven, as in the case of closed (i.e., full-flowing) conduits, or gravity driven. The latter refers to free-surface, or open-channel, flow in which the surface is exposed to the atmosphere. These systems tend to be more complicated from an analytical perspective since the surface is not constrained and flow depth is a function of discharge and the slope and shape of the channel. The majority of storm water and wastewater conveyance systems are designed to flow partly full, under open channel conditions, although smaller diameter, pressurized force mains may be used in some cases.

1.2.2 Steady vs. Unsteady Flow

Water and wastewater flows are classified as either steady or unsteady. Steady flows are those in which velocity, flow depth, and dependent terms at a particular location are constant over time. Unsteady conditions are characterized by temporal variations in flow velocity. In the majority of applications, flows can be modeled using the assumption of steady flow. Even if conditions change with time, those changes often occur slowly enough that the application of unsteady principles is unwarranted.

Example: Classify each of the following flows as steady or unsteady from an observer's viewpoint:

(a) Channel flow around bridge piers if the observer is

(1) stationary, standing on the bridge;
(2) in a drifting boat within the channel.

(b) A storm water surge through a sewer if the observer is

(1) stationary;
(2) moving with the surge.

Solution:

(a) Over short time periods, the stationary observer will see a steady flow, even though flow may be unsteady over longer time periods due to changes in quantity of runoff entering the channel. The observer in the boat will see an unsteady flow as the boat passes under the bridge because velocity increases around the piers (as the cross-sectional area of the channel is reduced), even if the approach conditions are steady.

(b) A stationary observer will see unsteady flow as the surge passes. A steady flow will be observed if the observer moves with the surge, assuming velocity is approximately constant over the distance considered.

1.2.3 Uniform vs. Non-uniform Flow

Uniform flow occurs when velocity, depth, and related properties are independent of location within a system; thus, this classification refers to the spatial variation of flow at a single instant in time. In open channels, flow will inevitably approach such conditions if the channel is prismatic (i.e., constant shape and slope) and is sufficiently long. Likewise, pressurized flow through a closed conduit having a constant diameter can be described as uniform. If conduit or channel geometry varies over distance, flow becomes non-uniform, or varied, which can be further classified as either gradually varied or rapidly varied depending on the rate of velocity change. In addition, spatially-varied flow is a particular type of gradually-varied flow in which discharge varies in the longitudinal flow direction due to lateral inflows or outflows to the primary flow system.

1.2.4 Newtonian vs. Non-Newtonian Flow

Viscosity was previously described as a parameter that represents a fluid's resistance to shear stress. For water and other thin liquids, viscosity remains constant and independent of the magnitude of shearing forces. These fluids are referred to as Newtonian fluids. In cases where viscosity varies, thus yielding a non-linear relationship between shear stress and deformation, fluids are characterized as non-Newtonian. Sludge, debris flow, and chemical slurries are common examples of the non-Newtonian fluids. In these cases, ample consideration must be given to the potentially-variable behavior of flow.

1.3 GEOMETRIC PROPERTIES

Application of hydraulic principles to water and wastewater systems requires the quantitative description of geometric elements for various conduits. For pressurized pipe flow, cross-sectional area, *A*, taken perpendicular to the primary flow direction, is a key expression. For a full-flowing, circular pipe of diameter *D*, area can be expressed as

$$A = \frac{\pi D^2}{4} \tag{1-9}$$

For open-channel systems, important geometric elements include cross-sectional area, wetted perimeter, *P*, top width of flow, *B*, and hydraulic radius, *R*. Wetted perimeter is defined as the portion of the conduit perimeter that is wetted by the fluid; whereas the hydraulic radius is the ratio of cross-sectional area to wetted perimeter (i.e., *A*/*P*). The hydraulic radius can also be used to evaluate an equivalent diameter when evaluating flow through pressurized, non-circular pipes. Table 1-2 lists a series of formulae that can be used to compute these various parameters.

It is worth noting that a large majority of pipes and sewers constructed in the United States in recent history are circular in cross section. Previously, however, a wide variety of irregular sections were common, including egg-shaped, semi-elliptical, horseshoe, oval, and others. Since circular sections have become far more commercially available and are subjected to significantly improved finishing techniques, circular sections have now become a standard cross-sectional shape.

Example: Develop a general expression for the equivalent diameter of full-flowing, non-circular conduits.

Solution:

The hydraulic radius for a full-flowing pipe is

$$R = \frac{A}{P} = \frac{\pi d^2/4}{\pi d} = \frac{D}{4}$$

Thus, *D* can be generally expressed as *4R* for non-circular systems.

Example: A 3-m wide rectangular channel flows at a depth of 0.5 m. Evaluate the area and hydraulic radius.

Solution:

Using formulae from Table 1-2, area and wetted perimeter are

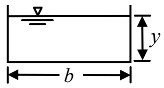
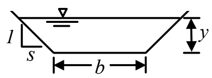
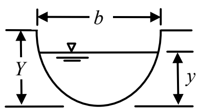
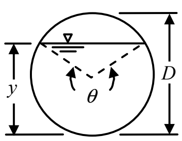
$$A = by = (3\ m)(0.5\ m) = 1.5\ m^2$$

$$P = b + 2y = 3\ m + (2)(0.5\ m) = 4\ m$$

and hydraulic radius is expressed as

$$R = \frac{A}{P} = \frac{1.5\ m^2}{4\ m} = 0.38\ m$$

Table 1-2: Key geometric properties

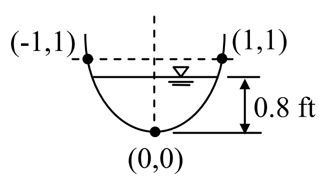
Shape	Area (A)	Wetted Perimeter (P)	Top width (B)	Hydraulic radius (R)
Rectangular 	by	$b+2y$	b	$\frac{by}{b+2y}$
Trapezoidal[a] 	$(b+sy)y$	$b+2y\sqrt{1+s^2}$	$b+2sy$	$\frac{(b+sy)y}{b+2y\sqrt{1+s^2}}$
Parabolic[b] 	$\frac{2}{3}By$	$\frac{B}{2}\left[\sqrt{1+x^2}+\left(x^{-1}\right)\ln\left(x+\sqrt{1+x^2}\right)\right]$	$b\sqrt{\frac{y}{Y}}$	$\frac{4y}{3}\left[\sqrt{1+x^2}+\left(x^{-1}\right)\ln\left(x+\sqrt{1+x^2}\right)\right]^{-1}$
Circular[c] 	$\frac{(\theta-\sin\theta)D^2}{8}$	$\frac{\theta D}{2}$	$D\sin\frac{\theta}{2}$ or $2\sqrt{y(D-y)}$	$\frac{D}{4}\left(1-\frac{\sin\theta}{\theta}\right)$

[a] For triangular sections, use $b = 0$

[b] $x = 4y/B$

[c] Use θ in radians, where $\theta = 2\cos^{-1}\left(1-\frac{2y}{D}\right)$

Example: Evaluate the area and hydraulic radius of the parabolic channel shown below.



Solution:

Based on coordinates in the figure, b = 2.0 ft and Y = 1.0 ft. Then, top width and area are

$$B = b\sqrt{\frac{y}{Y}} = (2\ ft)\sqrt{\frac{0.8\ ft}{1\ ft}} = 1.8\ ft$$

$$A = \frac{2}{3}By = \frac{2}{3}(1.8\ ft)(0.8\ ft) = 1.0\ ft^2$$

Hydraulic radius is computed as

$$x = \frac{4y}{B} = \frac{4(0.8\ ft)}{1.8\ ft} = 1.8$$

$$R = \frac{4y}{3}\left[\sqrt{1+x^2} + \left(x^{-1}\right)ln\left(x+\sqrt{1+x^2}\right)\right]^{-1} =$$

$$\frac{4(0.8\,ft)}{3}\left[\sqrt{1+1.8^2} + \left(\frac{1}{1.8}\right)ln\left(1.8+\sqrt{1+1.8^2}\right)\right]^{-1} = 0.38\ ft$$

Example: Water flows at a depth of 18 in through a 30-in diameter pipe. Determine the area of flow and hydraulic radius.

Solution:

From the formulae in Table 1-2,

$$\theta = 2\cos^{-1}\left(1-\frac{2y}{D}\right) = 2\cos^{-1}\left(1-\frac{2\left(18\,in\times\frac{1\ ft}{12\,in}\right)}{2.5\ ft}\right) = 3.54\ rad$$

$$A = \frac{(\theta-\sin\theta)D^2}{8} = \frac{(3.54-\sin 3.54)(2.5\ ft)^2}{8} = 3.1\ ft^2$$

$$R = \frac{D}{4}\left(1-\frac{\sin\theta}{\theta}\right) = \frac{(2.5\ ft)}{4}\left(1-\frac{\sin 3.54}{3.54}\right) = 0.7\ ft$$

1.4 FLOW RATE

Volumetric flow, or discharge, Q, through a pipe or channel refers to the rate at which a given volume of fluid passes a fixed point (i.e., $\Delta\forall/\Delta t$). In most cases, it can be more conveniently expressed as the product of mean flow velocity, V, and cross-sectional area of flow. Typical units are ft^3/s (cfs) and m^3/s. For a constant-density fluid, the mass rate of flow past a fixed point, $\dot{m}$ (i.e., $\Delta m/\Delta t$), can be expressed as the product of mass density and discharge.

<u>Example</u>: Water flows through a 6-in pipe at an average velocity of 4 ft/s (fps). Evaluate the volumetric and mass flow rate.

Solution:

The cross-sectional area of the circular pipe is computed as

$$A = \frac{\pi D^2}{4} = \frac{(3.14)\left(6\ in \times \frac{1\ ft}{12\ in}\right)^2}{4} = 0.20\ ft^2$$

$$Q = VA = (4\ ft/s)(0.20\ ft^2) = 0.8\ cfs$$

$$\dot{m} = \rho Q = (1.94\ slugs/ft^3)(0.8\ cfs) = 1.6\ slugs/s$$

<u>Example</u>: Flow travels through a trapezoidal channel having side slopes of 2H:1V and a bottom width of 2 ft. If the depth is 0.75 ft, compute the flow area, hydraulic radius, and mean flow velocity for a discharge of 3 cfs.

Solution:

Noting that $y = 0.75$ ft, $b = 2$ ft, and $s = 2$,

$$A = (b + sy)y = [2\ ft + (2 \times 0.75\ ft)]0.75\ ft = 2.63\ ft^2$$

$$R = \frac{(b + sy)y}{b + 2y\sqrt{1 + s^2}} = \frac{[2\ ft + (2 \times 0.75\ ft)]0.75\ ft}{2\ ft + (2 \times 0.75\ ft)\sqrt{1 + 2^2}} = 0.49\ ft$$

Since $Q = VA$, the mean flow velocity is evaluated as

$$V = \frac{Q}{A} = \frac{3\ cfs}{2.63\ ft^2} = 1.14\ fps$$

1.5 GENERALIZED EQUATIONS OF STEADY FLOW

1.5.1 Conservation of Mass

The continuity equation for steady, constant-density flow implies that discharge at different pipe or channel sections remains equal, provided there is no lateral inflow or outflow over the length being considered. Referring to Figure 1-1, the continuity equation is expressed as

$$Q_1 = Q_2 \tag{1-10}$$

or

$$V_1 A_1 = V_2 A_2 \tag{1-11}$$

<u>Example</u>: Pressurized flow travels through a 12-in pipe at 8 fps. If the pipe expands to a diameter of 18 in, what is velocity in the larger section?

Solution:

From Equation 1-11,

$$V_{18} = V_{12}\frac{A_{12}}{A_{18}} = V_{12}\left(\frac{\pi D_{12}^2/4}{\pi D_{18}^2/4}\right) =$$

$$V_{12}\left(\frac{D_{12}}{D_{18}}\right)^2 = 8\ fps\left(\frac{12\ in}{18\ in}\right)^2 = 3.56\ fps$$

<u>Example</u>: A trapezoidal channel has a bottom width of 1.5 m and side slopes of 2H:1V. If the velocity and depth at an upstream location are 1 m/s and 0.25 m, determine the velocity at a downstream section where the depth is measured as 0.35 m.

Solution:

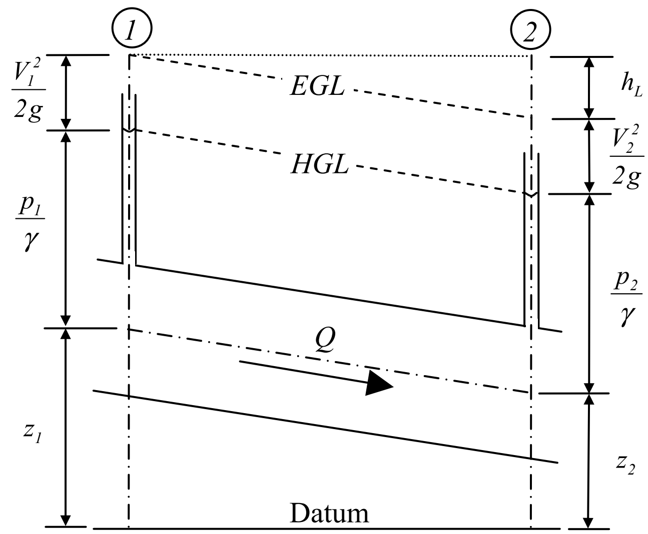
Downstream and upstream flow areas are computed as

$$A_d = (b + sy)y = [1.5\ m + (2 \times 0.35\ m)]0.35\ m = 0.77\ m^2$$

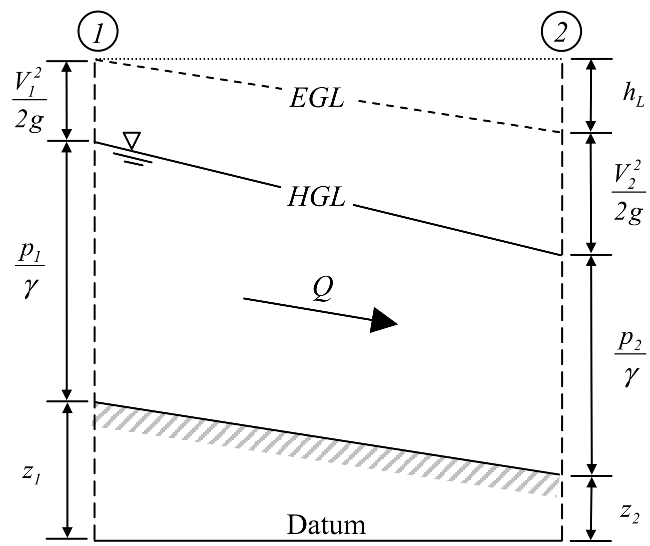
$$A_u = [1.5\ m + (2 \times 0.25\ m)]0.25\ m = 0.50\ m^2$$

The mean velocity at the upstream section is then

$$V_u = V_d\frac{A_d}{A_u} = 1\ m/s\left(\frac{0.77\ m^2}{0.50\ m^2}\right) = 1.54\ m/s$$



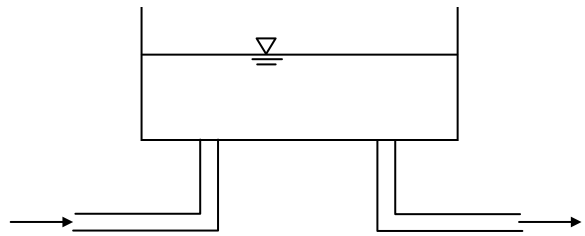
(a)



(b)

Figure 1-1: Definition sketch for (a) pipe flow and (b) open channel flow

Example: Water flows into a 6-m diameter, cylindrical tank at 0.05 m^3/s and exits through a 150-mm diameter pipe. If the level in the tank rises at a rate of 0.8 mm/s, determine the outlet velocity in the pipe.



Solution:

Although similar to previous examples, the discharge associated with rising fluid in the cylindrical tank must be accounted for as an additional outflow. This discharge is computed as

$$Q_{tank} = A_{tank}\frac{dh}{dt} = \left[\frac{\pi(6\,m)^2}{4}\right]0.8\times10^{-3}\ m/s = 0.023 m^3/s$$

Thus, total outflow is the sum of discharge in the 150-mm pipe and that in the tank, and

$$Q_{in} = V_{out}A_{out} + Q_{tank} = 0.05\,m^3/s = V_2\left[\frac{\pi(0.15\,m)^2}{4}\right] + 0.023$$

Solving for V_{out} yields a velocity of 1.5 m/s.

1.5.2 Conservation of Energy

Conservation of energy involves a balance in total energy, expressed as head, between any upstream point of flow and a corresponding downstream point, including energy, or head, losses. These losses are caused by friction and the viscous dissipation of turbulence at bends and other appurtenances (i.e., form losses). The energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_L \qquad (1\text{-}12)$$

where the subscripts *1* and *2* refer to upstream and downstream sections, respectively; p/γ is referred to as pressure head; z is elevation of the pipe centerline for pressurized systems or the channel invert for gravity systems, both with respect to a specified datum; $V^2/2g$ is referred to as velocity head; h_p is head supplied by a pump, if one exists; and h_L represents friction and form losses. The sum of pressure head and elevation terms at any one section defines the location of the hydraulic grade line (HGL), while the addition of velocity head to the HGL describes the energy grade line (EGL).

The Coriolis coefficient, α, also known as the kinetic energy correction factor, in Equation 1-12 is used to account for the effects of a non-uniform velocity distribution. It is computed by

$$\alpha = \frac{1}{V^3 A}\int_A u^3 dA \qquad (1\text{-}13)$$

where u is the velocity at any point in a cross section. It can be shown that α assumes a value ranging from unity, when velocity is uniform across a pipe section (e.g., turbulent flow), to 2.0, when the velocity distribution is highly non-uniform (e.g., laminar flow). In the majority of practical cases, a value of 1.0 can be safely assumed.

Example: Water is pumped from an open tank and through a 30-cm diameter pipe at a rate of 0.25 m^3/s. At a point in the system where the pipe lies 10 m above the elevation of the water surface in the tank, the pressure is measured as 100 kPa (gage). If head losses are estimated as 3.0 × $V^2/2g$, where V is the average velocity in the pipe, compute the energy added to the system by the pump.

Solution:

Here, *1* is at the water surface in the tank and *2* is in the 30-cm diameter pipe. Velocity in the pipe is computed as

$$V_2 = \frac{Q}{A_2} = \frac{0.25\ m^3/s}{\left(\dfrac{\pi(0.30\ m)^2}{4}\right)} = 3.5\ m/s$$

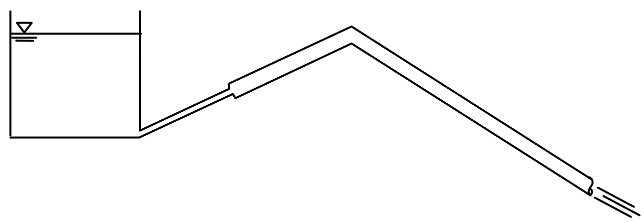
Assuming a Coriolis coefficient of 1.0, the energy equation is written between the surface of the tank and the point of interest in the pipe as follows:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L=$$

$$0+0+0+h_p=\frac{100{,}000\ Pa}{9{,}810\ N/m^3}+\frac{(3.5\ m/s)^2}{2\times 9.81\ m/s^2}+10\ m+\frac{(3.0)(3.5\ m/s)^2}{2\times 9.81\ m/s^2}$$

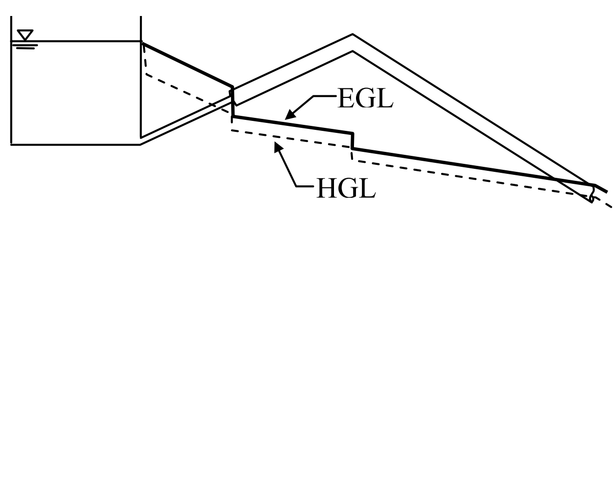
Solving for h_p yields a value of 22.7 m (i.e., 22.7 m of head is added by the pump).

Example: Sketch HGL and EGL for the pipe system shown below.



Solution:

The HGL and EGL are drawn below, along with the pipe system. Notable features include the (a) form losses that occur at the pipe entrance, the abrupt expansion, and the bend all cause drops in the HGL and EGL; (b) a decreasing HGL and EGL slope is caused by a reduced velocity head in the region corresponding to the larger diameter pipe; (c) the convergence of the HGL to the free jet, indicating atmospheric pressure (i.e., 0 gage) at the pipe exit; and (d) the elevation of the HGL between the expansion and the free jet lies below the pipe. The last feature is indicative of a problem with the current system design in that a subatmospheric condition exists in the latter two-thirds of the pipe as a combined result of pipe elevation change and head loss.



1.5.3 Conservation of Momentum

Conservation of momentum relates applied forces to changes in linear momentum, which are evidenced by changes in direction or magnitude of flow velocity (i.e., non-uniform flow). Relevant forces include those transmitted to the fluid from pipe or channel walls and those originating from the pressure variation through the fluid. For any direction *s*, the momentum equation for steady flow can be written as

$$\sum F_s = \beta \rho Q \left(V_{2,s} - V_{1,s} \right) \qquad (1\text{-}14)$$

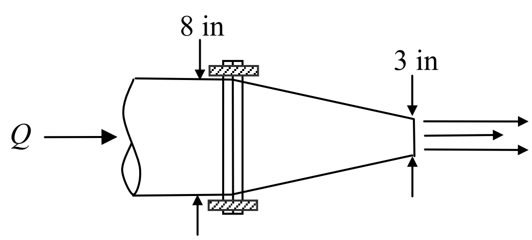
where $\sum F_s$ is the resultant of forces in the *s* direction, including hydrostatic pressure forces (i.e., pressure multiplied by flow area), gravity forces, and other externally applied forces; and $V_{2,s}$ and $V_{1,s}$ refer to mean flow velocities in the *s* direction at downstream and upstream cross sections, respectively.

The Boussinesq coefficient, β, also known as the momentum correction factor, is used to account for the effects on momentum of a non-uniform velocity distribution at a particular pipe section. It can be evaluated as

$$\beta = \frac{1}{V^2 A} \int_A u^2 dA \qquad (1\text{-}15)$$

As with the Coriolis coefficient, for most applications of practical interest, the velocity distribution can be approximated as uniform, and the coefficient assumes a value of 1.0.

<u>*Example*</u>*:* Water flows through the nozzle shown below. The pressure at the nozzle inlet is 80 psi (gage), and the exit velocity is 72 fps. Determine the force provided by the flange bolts in order keep the nozzle stationary.



Solution:

The inlet velocity is computed using continuity, expressed as

$$V_{in} = V_{out} \frac{A_{out}}{A_{in}} = V_{out} \left(\frac{D_{out}}{D_{in}} \right)^2 = 72\ fps \left(\frac{3\ in}{8\ in} \right)^2 = 10.1\ fps$$

and discharge is evaluated as

$$Q = V_{out} A_{out} = 72\ fps \left(\frac{\pi \left(3\ in \times \frac{1\ ft}{12\ in} \right)^2}{4} \right) = 3.5\ cfs$$

Assuming β = 1.0 and that the force of the bolts acts to the left, the momentum equation can be written in the primary flow direction as

$$p_1 A_1 - F_{bolts} = \rho Q (V_2 - V_1)$$

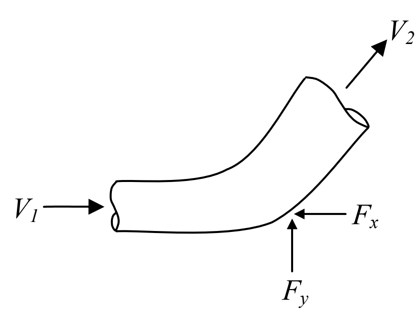
The first term in the equation represents the hydrostatic pressure component in the pipe. This force acts in the direction of flow at the inlet and is zero at the outlet, where pressure is zero gage. The unknown force can thus be evaluated as

$$F_{bolts} = p_1 A_1 - \rho Q (V_2 - V_1) =$$

$$\left(80\ psi \times 144\ in^2 / ft^2 \right) \left(\frac{\pi \left(8\ in \times \frac{1\ ft}{12\ in} \right)^2}{4} \right) -$$

$$\left(1.94\ slugs / ft^3 \right) (3.5\ cfs)(72\ fps - 10.1\ fps) = 3{,}600\ lbs$$

<u>Example</u>*:* A pipe system contains a 30° horizontal expansion bend, as shown below, that transitions from 6-in diameter to 12-in diameter. The total head loss through the bend is 2 ft, and the pipe carries a discharge of 2 cfs. If the pressure at the inlet to the bend is 25 psi (gage), find the resultant horizontal forces required to stabilize the bend.



Solution:

The upstream and downstream velocities are

$$V_1 = \frac{Q}{A_1} = \frac{2\ cfs}{\left(\frac{\pi (0.5\ ft)^2}{4}\right)} = 10.2\ fps$$

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D_1}{D_2}\right)^2 = 10.2\ fps \left(\frac{6\ in}{12\ in}\right)^2 = 2.6\ fps$$

Noting that this is a horizontal bend (i.e., no elevation change) and assuming $\alpha = 1.0$, the energy equation can simplifies to

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

or

$$p_2 = \gamma \left(\frac{p_1}{\gamma} + \frac{V_1^2 - V_2^2}{2g} - h_L\right) =$$

$$62.4\ lbs/ft^3 \left(\frac{25\ psi \times 144\ in^2/ft^2}{62.4\ lbs/ft^3} + \frac{(10.2\ fps)^2 - (2.6\ fps)^2}{2 \times 32.2\ ft/s^2} - 2\ ft\right)$$

Solving for the exit pressure yields p_2 = 3,570 psf. The momentum equation is applied in both the x and y directions to determine component forces. For the x direction, assuming a Boussinesq coefficient equal to 1.0,

$$p_1 A_1 - \cos 30\, p_2 A_2 - F_x = \rho Q (V_2 \cos 30 - V_1)$$

Note the hydrostatic pressure forces (i.e., first two terms on the left-hand side of the equation) act on the pipe bend. Thus, the force acts to the right at the inlet and downward to the left at the outlet. Substituting values yields

$$(25\ psi \times 144\ in^2/ft^2)\left(\frac{\pi (0.5\ ft)^2}{4}\right)$$

$$-(\cos 30)(3{,}570\ pf)\left(\frac{\pi (1\ ft)^2}{4}\right) - F_x =$$

$$(1.94\ slugs/ft^3 \times 2\ cfs)[(2.6\ fps)(\cos 30) - 10.2\ fps]$$

Solving for F_x yields -1,690 lbs. The negative sign indicates that the wrong direction was initially assumed; thus, the force F_x instead acts to the right. Similarly, for the y direction,

$$F_y - \sin 30\, p_2 A_2 = \rho Q\left[V_2 \sin 30 - 0\right]$$

Note that since there is no flow in the y direction at the inlet, there is no velocity or hydrostatic force in the y direction at that location.

$$F_y - (\sin 30)(3{,}570\ psf)\left(\frac{\pi(1\ ft)^2}{4}\right) =$$
$$+\left(1.94\ slugs/ft^3 \times 2\ cfs\right)\left[(2.6\ fps)(\sin 30) - 0\right]$$

Solving for F_y yields 1,410 lbs. The assumed direction was correct (i.e., positive number) and F_y acts upward. The resultant force is computed as

$$F_R = \sqrt{(1{,}690\ lbs)^2 + (1{,}410\ lbs)^2} = 2{,}200\ \ lbs$$

which acts upward to the right at an angle of

$$\tan^{-1}\left(\frac{1{,}410\ lbs}{1{,}690\ lbs}\right) = 40^\circ \angle$$